

INVARIANT PROPERTIES OF
A SHEET ASPHALT MIXTURE

JANUARY 1966

NO. 1

by

N.B. LAL

W.H. GOETZ

M.E. HARR

Joint
Highway
Research
Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

INVARIANT PROPERTIES

OF A

SHEET ASPHALT MIXTURE

by

N. B. Lal

Assistant Professor of Civil Engineering
Punjab Engineering College - formerly
Graduate Assistant, Purdue University

W. H. Goetz

Professor of Highway Engineering
Purdue University

M. E. Harr

Professor of Soil Mechanics
Purdue University

for

Presentation to 45th Annual Meeting of
Highway Research Board, January 17-21, 1966

Purdue University
Lafayette, Indiana
January 1966

55.

251072 251073 251074

1. The first of these is the fact that the
2. second of these is the fact that the
3. third of these is the fact that the
4. fourth of these is the fact that the
5. fifth of these is the fact that the

[illegible]

March 11, 1944

55.

[illegible]

1980年1月
 1980年1月
 1980年1月

ABRIDGMENT

Lal, N. B., Goetz, W. H. and Harr, M. E., "Invariant Properties of a Sheet Asphalt Mixture". Presented at the 45th Annual Meeting of the Highway Research Board, Washington, D. C., January 1966. Manuscript copy consists of 13 pages, 3 tables, 11 figures.

Descriptors: sheet asphalt, tension test, shear test, hollow cylinder, compression test, stress-strain-temperature relationships.

The study was undertaken to provide invariant properties of a sheet asphalt mixture. To do this three independent equations were sought which in combination with the two general two-dimensional equations of motion, involving five unknowns, will render the system complete.

The required three equations were determined on the basis of experimental data obtained from two different types of laboratory tests. Uniaxial Tension and Simple Shear tests were chosen for this purpose.

The Uniaxial Tension tests were performed on cylindrical specimens by subjecting them to constant stress at constant temperature and observing the axial and circumferential strains with time. The tests were repeated under different stresses at three temperatures. On the basis of the data obtained from these tests, an equation relating stress (σ_z) to axial strain (ϵ_z) was derived. The equation had the following form:

$$\sigma_z = \left[\frac{T}{c_1} \right]^{-c_2} - \left[\frac{T}{p_1} \right]^{-p_2} \frac{\epsilon_z}{t \frac{\partial \epsilon_z}{\partial t}} ; 40^\circ\text{F} \leq T \leq 100^\circ\text{F}$$

where t stands for time, T for temperature and c_1 , c_2 , p_1 , p_2 are four material constants. A similar expression relating stress (σ_z) to circumferential strain (ϵ_y) was also obtained from the results of these tests.

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation $f(x) = \sum_{n=0}^{\infty} a_n x^n$, where a_n are the coefficients of the power series. It is shown that $f(x)$ is a continuous function of x and that it satisfies the functional equation $f(x) = f(x^2) + x f(x)$. The second part of the paper is devoted to the study of the properties of the function $g(x)$ defined by the equation $g(x) = \sum_{n=0}^{\infty} b_n x^n$, where b_n are the coefficients of the power series. It is shown that $g(x)$ is a continuous function of x and that it satisfies the functional equation $g(x) = g(x^2) + x g(x)$.

The third part of the paper is devoted to the study of the properties of the function $h(x)$ defined by the equation $h(x) = \sum_{n=0}^{\infty} c_n x^n$, where c_n are the coefficients of the power series. It is shown that $h(x)$ is a continuous function of x and that it satisfies the functional equation $h(x) = h(x^2) + x h(x)$. The fourth part of the paper is devoted to the study of the properties of the function $k(x)$ defined by the equation $k(x) = \sum_{n=0}^{\infty} d_n x^n$, where d_n are the coefficients of the power series. It is shown that $k(x)$ is a continuous function of x and that it satisfies the functional equation $k(x) = k(x^2) + x k(x)$.

$$\frac{1}{x} \frac{d}{dx} \left(x \frac{d}{dx} \right) f(x) = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} n a_n x^{n-1}$$

The fifth part of the paper is devoted to the study of the properties of the function $l(x)$ defined by the equation $l(x) = \sum_{n=0}^{\infty} e_n x^n$, where e_n are the coefficients of the power series. It is shown that $l(x)$ is a continuous function of x and that it satisfies the functional equation $l(x) = l(x^2) + x l(x)$. The sixth part of the paper is devoted to the study of the properties of the function $m(x)$ defined by the equation $m(x) = \sum_{n=0}^{\infty} f_n x^n$, where f_n are the coefficients of the power series. It is shown that $m(x)$ is a continuous function of x and that it satisfies the functional equation $m(x) = m(x^2) + x m(x)$.

The Simple Shear tests were performed on thin rectangular specimens by subjecting them to constant shear stress at constant temperature and observing the shear strain with time. The tests were repeated under different stresses at three temperatures. On the basis of these tests, an expression relating shear stress to shear strain was obtained in the same form as given above for Uniaxial Tension tests.

The four material constants as found from the stress-strain expressions derived from the above two types of tests were independent of time and temperature for small values of strain. Since the values of these material constants as determined from the two series of tests were in close agreement, it was indicated that these are also independent of the type of test.

Axial Compression tests were performed on hollow cylindrical specimens to compare the results with those predicted on the basis of the corresponding Uniaxial Tension tests. It was found that for small strains of less than about 0.4 percent, the two tests gave very close results. For large strains, whereas the strain (ordinate)-time (abscissa) plot on log-log scales tended to curve upward at the beginning of failure conditions in Uniaxial Tension tests, the corresponding plots for Axial Compression tests tended to curve downward to lesser slopes, at about the same time.

It was concluded that three independent stress-strain relationships exist as functions of time and temperature. These expressions contain four basic material constants which are independent of time and temperature and type of test.

SYNOPSIS

In this work properties of a sheet asphalt mixture are obtained that are believed to have greater quantitative significance than those usually used to describe bituminous mixtures. The results are based upon Newton's equations of motion under conditions of plane strain. As these equations are two in number but contain five unknowns, laboratory tests were conducted to obtain three additional independent expressions relating the unknowns. Only those parts of the relationships that were found to be reproducible and independent of time, temperature and conditions of test were considered material properties. Four basic material constants were obtained as opposed to the more usual three constants: modulus of elasticity, Poisson's ratio, and coefficient of thermal expansion.

INTRODUCTION

Newton's equations of motion for a two-dimensional system are:

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} = \gamma + \rho \frac{\partial^2 w}{\partial t^2}; \quad \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \sigma_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \quad (1)$$

where σ_z and σ_y are normal stresses and w and v are displacements in the vertical (z) and horizontal (y) directions, respectively; τ_{yz} is the shear stress in the plane under consideration; γ is the unit weight of the mixture, and ρ is its mass unit weight ($\rho = \gamma/g$).

Equations (1) contain five unknowns: σ_z , σ_y , τ_{yz} , w , v . Hence, three more independent expressions relating these are needed to render the system solvable. To achieve this balance, recourse was made to simple laboratory tests conducted under inputs varying with time and temperature wherein pertinent relationships may be obtained from relevant observations.

the first of these is the fact that the system is not in equilibrium.

The second is the fact that the system is not in equilibrium.

The third is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

The second is the fact that the system is not in equilibrium.

The third is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

The second is the fact that the system is not in equilibrium.

The third is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

The second is the fact that the system is not in equilibrium.

The third is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

The second is the fact that the system is not in equilibrium.

The third is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

the first of these is the fact that the system is not in equilibrium.

Any parameters relating the unknowns that are found to be independent of type of test or conditions of loading are material properties of the bituminous mixture studied. Underlying this concept is the requirement that for any material property to have quantitative significance it must, of necessity, reflect the action of the material "in situ". Hence, true properties must remain unchanged (invariant) under transformations from laboratory to field conditions. For properties to be invariant under transformations from a simple laboratory test to very complicated field conditions, they must of necessity also remain constant under different laboratory tests. Using this condition of necessity in combination with Eqns. (1) and conducting different types of simple laboratory tests, with known boundary conditions, those parameters remaining constant were duly noted as material properties.

METHODS OF TESTING

In order to obtain the three equations required for solving the aforementioned two-dimensional deformable system, relevant experimental data had to be obtained from different types of tests. For this purpose Uniaxial Tension and Simple Shear tests were chosen. To verify the material constants as obtained from these tests, an Axial Compression test was utilized. The considerations for the choice of these tests and the experimental techniques employed therein are given below under separate headings for each.

Uniaxial Tension Tests

To evaluate the material constants expected to be found in the relationships between normal stress and strains along and at right angles to the direction of application of load, a Uniaxial Tension test was performed. This was done because in such a test the shear stresses are zero in these directions and the material may be considered as subject to normal tensile stresses only.

A cylindrical specimen two-inches in diameter and four-inches high was subjected to constant tensile stress at constant temperature and the elongation per inch as well as decrease in radius per inch was noted with time. To reduce the end-effects, the measurements of strain were confined to the middle portion of the specimen. Also, to minimize the errors due to any possible eccentricity during the application of load, the strains were measured at locations 180° apart in the middle portion of the specimen. In this way, by measuring the elongation of the middle one-inch portion and the reduction in diameter at the middle, plots of axial strain versus time and lateral strain versus time, at constant tensile stress, were obtained. By repeating the above procedures for different temperatures, the relationships between normal stress and the axial and lateral strains were obtained as functions of time and temperature.

Figure 1 shows the instrumentation for this test. A simple mechanical device was developed which greatly enhanced the strain measuring technique. This consisted of levers with pointed ends which made contact with the specimen. Deformation of the specimen at the point of contact with the pointed end of the lever was recorded by a dial indicator attached to the lever end away from the specimen. Axial strain in the middle portion of the specimen was determined by the difference in deformations recorded by dial indicators attached to the levers 1 inch apart vertically. The readings of the dial indicators were

estimated up to 0.00005 inch. Change in diameter at mid-height of specimen was recorded by dial indicators with their extensions, machined to fit the curved surface, resting directly on the specimen surface.

Simple Shear Tests

The relationship between shear stress and shear strain was determined from a Simple Shear test wherein the normal stresses may be taken to be zero in the considered planes.

A simple and direct means of determining shear strain as a function of time under constant stress and at constant temperature was achieved by forming a specimen of appropriate thickness, fixing one face and pulling the other parallel face under constant load. A diagrammatic sketch of this test is shown in Figure 2. In deciding upon the thickness of specimens for these Simple Shear tests, the following points were considered:

1. Minimum amount of bending while the specimen is being subjected to simple shearing stress.
2. Non-interference of particles within the specimen.
3. Practicability of fabricating specimens with uniformity or homogeneity of compacted materials.
4. Ability to record the dilation of the specimen while undergoing shearing strain.

After trying several thicknesses with the above points considered, a thickness of 1/4 inch and a specimen size of 4 x 2 inches were chosen.

Axial Compression Tests

A hollow-cylinder compression test was performed in an attempt to verify the material constants as obtained from the Uniaxial Tension and Simple Shear tests. In this test, deformations of the material were observed both in the axial as well as the lateral direction by noting the deformations on the inside

as well as the outside of the hollow cylinder. Again, the tests were performed under constant load and at constant temperature. The instrumentation developed for measuring axial deformations and change in external diameter in the Uniaxial Tension tests was applicable for these tests also. For measuring change in inside diameter, a modification of this lever system using dial indicators was used. This is shown by the diagrammatic sketch of Figure 3.

The hollow cylindrical specimen, having a 2-inch external diameter, 1-inch internal diameter and 4-inch height, was placed on a hollow steel cylinder fitted with two hinged levers. The upper parts of the hinges were machined to correspond to the inside curved surface of the specimen and the lower ends were contacted by extensions of the dial indicators. Changes in the diameter of the hole were recorded with time. Changes in outside diameter were recorded with time by dial indicators resting directly on the surface. Changes in unit thickness of specimen, or lateral strain, were thus calculated from the difference in changes of outside and inside diameters.

SUMMARY OF TEST RESULTS

To obtain the experimental data required for this study, Uniaxial Tension, Simple Shear and Axial Compression tests were performed. In this order, the results of these tests are summarized here. Complete test results for Uniaxial Tension and Simple Shear tests are tabulated, but only the data for tests at 77°F are presented in plot form, including those for the axial compression tests.

The test results for the Uniaxial Tension tests are given in Tables 1 and 2 for axial and circumferential strain, respectively. As can be seen from Figure 4, the axial strain of specimens under constant stress and constant temperature, when plotted as ordinate against time on log-log scales gave a straight line relation-

ship in the pre-failure region. As failure started to take place with the appearance of minute cracks in the middle portion of specimen, the straight line on the log-log plot tended to curve upward. It was found convenient to characterize the straight line portion of the plot by its slope and axial strain at one minute (it being a log scale). Within the range of temperatures and stress-levels tested, the following points of interest were observed from the Uniaxial Tension test results:

At constant temperature, the axial strain at one minute did not vary proportionally with applied stress. The deviation from proportionality increased with increasing temperature as well as with increasing applied stress. The slopes of the straight-line portions of the log-log plots varied with the applied stress and temperature. The slopes became steeper with increase in applied stress at constant temperature. For an incremental change in stress, the corresponding change in slope was greater at higher temperatures.

The circumferential strain when plotted as ordinate against time on log-log scales also gave a straight-line relationship. See Figure 5. The same trends as observed for axial strains were observed for circumferential strains.

A study of Poisson's ratio for the material as determined from the Uniaxial Tension test data showed it to be a function of applied stress, time and temperature. Whereas at 40°F its value was found to be almost independent of applied stress, as would be the case for an elastic material, at higher temperatures it decreased with increasing applied stress.

Results for the Simple Shear tests are given in Table 3. As can be seen from Figure 6, the shearing strain in the specimen under constant shear stress and constant temperature, when plotted as ordinate against time on log-log scales, gave a straight-line relationship in the pre-failure region. The shearing strain-

the first of these is the fact that the
the second is the fact that the
the third is the fact that the
the fourth is the fact that the
the fifth is the fact that the
the sixth is the fact that the
the seventh is the fact that the
the eighth is the fact that the
the ninth is the fact that the
the tenth is the fact that the
the eleventh is the fact that the
the twelfth is the fact that the
the thirteenth is the fact that the
the fourteenth is the fact that the
the fifteenth is the fact that the
the sixteenth is the fact that the
the seventeenth is the fact that the
the eighteenth is the fact that the
the nineteenth is the fact that the
the twentieth is the fact that the
the twenty-first is the fact that the
the twenty-second is the fact that the
the twenty-third is the fact that the
the twenty-fourth is the fact that the
the twenty-fifth is the fact that the
the twenty-sixth is the fact that the
the twenty-seventh is the fact that the
the twenty-eighth is the fact that the
the twenty-ninth is the fact that the
the thirtieth is the fact that the
the thirty-first is the fact that the
the thirty-second is the fact that the
the thirty-third is the fact that the
the thirty-fourth is the fact that the
the thirty-fifth is the fact that the
the thirty-sixth is the fact that the
the thirty-seventh is the fact that the
the thirty-eighth is the fact that the
the thirty-ninth is the fact that the
the fortieth is the fact that the
the forty-first is the fact that the
the forty-second is the fact that the
the forty-third is the fact that the
the forty-fourth is the fact that the
the forty-fifth is the fact that the
the forty-sixth is the fact that the
the forty-seventh is the fact that the
the forty-eighth is the fact that the
the forty-ninth is the fact that the
the fiftieth is the fact that the
the fifty-first is the fact that the
the fifty-second is the fact that the
the fifty-third is the fact that the
the fifty-fourth is the fact that the
the fifty-fifth is the fact that the
the fifty-sixth is the fact that the
the fifty-seventh is the fact that the
the fifty-eighth is the fact that the
the fifty-ninth is the fact that the
the sixtieth is the fact that the
the sixty-first is the fact that the
the sixty-second is the fact that the
the sixty-third is the fact that the
the sixty-fourth is the fact that the
the sixty-fifth is the fact that the
the sixty-sixth is the fact that the
the sixty-seventh is the fact that the
the sixty-eighth is the fact that the
the sixty-ninth is the fact that the
the seventieth is the fact that the
the seventy-first is the fact that the
the seventy-second is the fact that the
the seventy-third is the fact that the
the seventy-fourth is the fact that the
the seventy-fifth is the fact that the
the seventy-sixth is the fact that the
the seventy-seventh is the fact that the
the seventy-eighth is the fact that the
the seventy-ninth is the fact that the
the eightieth is the fact that the
the eighty-first is the fact that the
the eighty-second is the fact that the
the eighty-third is the fact that the
the eighty-fourth is the fact that the
the eighty-fifth is the fact that the
the eighty-sixth is the fact that the
the eighty-seventh is the fact that the
the eighty-eighth is the fact that the
the eighty-ninth is the fact that the
the ninetieth is the fact that the
the ninety-first is the fact that the
the ninety-second is the fact that the
the ninety-third is the fact that the
the ninety-fourth is the fact that the
the ninety-fifth is the fact that the
the ninety-sixth is the fact that the
the ninety-seventh is the fact that the
the ninety-eighth is the fact that the
the ninety-ninth is the fact that the
the hundredth is the fact that the

time relationships showed the same trends with regard to temperature and applied stress as were observed in the axial strain-time relationships as determined by the Uniaxial Tension tests. This was an indication of the fact that the same basic material properties were being reflected in these two types of tests.

For the Axial Compression tests, the log-log plots of axial strain vs. time (Figure 7) were found not to be continuous straight lines for the entire range. The plots were straight lines up to a certain percentage of deformation after which they curved downward to lesser slopes. As shown in Figure 8, a similar result was found for circumferential strains recorded in the axial compression tests. The initial straight-line portions of the Axial Compression test plots showed the same trends with regard to applied stress and temperature as were observed in the axial strain-time plots from the Uniaxial Tension tests. It was also observed from the Axial Compression test results that strains up to about 0.4 percent can be quite satisfactorily predicted from the Uniaxial Tension tests.

DERIVATION OF STRESS-STRAIN EXPRESSIONS

The derivations of stress-strain expressions from Uniaxial Tension and Simple Shear test results are based on the following observations:

- 1) The strain-time plots in the pre-failure regions on log-log scales were straight lines.
- 2) The slopes of these straight lines varied with the applied stress and temperature.

Derivation of the relationship between normal stress σ_z and axial strain ϵ_z as a function of time and temperature is as follows. Axial strain-time plots on log-log scales being straight lines in pre-failure regions can be represented as:

... ..
... ..
... ..
... ..
... ..

... ..
... ..
... ..
... ..
... ..

... ..
... ..
... ..
... ..
... ..

... ..
... ..
... ..
... ..
... ..

... ..
... ..
... ..
... ..
... ..

... ..
... ..

$\log \epsilon_z = \log k_1 + k_2 \log t$ where t stands for time, k_2 is the slope of the straight line and k_1 is the axial strain at unit time. Differentiating with respect to t , we get

$$\frac{1}{\epsilon_z} \frac{\partial \epsilon_z}{\partial t} = \frac{k_2}{t} \quad (1)$$

The slopes k_2 of these straight lines varied linearly with stress in the test range as shown in Figure 9 and hence yields the relation

$$\sigma_z = \frac{I(T)}{S(T)} - \frac{1}{S(T)} - \frac{1}{k_2} \quad (2)$$

where $I(T)$ and $S(T)$ are the intercepts on $1/k_2$ axis and slopes of the straight lines respectively as a function of temperature T .

Figure 10 shows on log-log scales a straight line relationship between temperature T and the ratio $\frac{I(T)}{S(T)}$ i.e., $\frac{I(T)}{S(T)} = \left[\frac{T}{c_1} \right]^{-c_2}$ where c_1 and c_2 are constants. Similarly, the log-log plot of Figure 11 shows $\frac{1}{S(T)} = \left[\frac{T}{p_1} \right]^{-p_2}$ where p_1 and p_2 are constants.

Substituting these values of $\frac{I(T)}{S(T)}$ and $\frac{1}{S(T)}$ and also the value of k_2 from (1) in equation (2), we get:

$$\sigma_z = \left[\frac{T}{c_1} \right]^{-c_2} - \left[\frac{T}{p_1} \right]^{-p_2} \frac{\epsilon_z}{t \frac{\partial \epsilon_z}{\partial t}}$$

where c_1, c_2, p_1, p_2 are material constants independent of time and temperature.

the first part of the paper, we consider the case where the system is in a steady state. In this case, the system is described by the following equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

where \mathbf{x} is the state vector, \mathbf{u} is the control vector, \mathbf{A} is the system matrix, and \mathbf{B} is the input matrix. The system is said to be in a steady state when the state vector \mathbf{x} is constant, i.e., $\frac{d\mathbf{x}}{dt} = 0$. In this case, the system is described by the following equation:

$$\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} = 0$$

where \mathbf{x} is the steady state state vector, and \mathbf{u} is the steady state control vector. The steady state state vector \mathbf{x} can be found by solving the following equation:

$$\mathbf{x} = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}$$

where \mathbf{A}^{-1} is the inverse of the system matrix \mathbf{A} . The steady state control vector \mathbf{u} can be found by solving the following equation:

DISCUSSION OF STRESS-STRAIN EXPRESSIONS

From Uniaxial Tension test results, the expression relating the normal tensile stress (σ_z) to axial strain (ϵ_z) was found to be:

$$\sigma_z = \left[\frac{T}{c_1} \right]^{-c_2} - \left[\frac{T}{p_1} \right]^{-p_2} t \frac{\epsilon_z}{\frac{\partial \epsilon_z}{\partial t}}$$

where t stands for time, T for temperature and c_1 , c_2 , p_1 , p_2 are material constants. An expression of the same form relating σ_z to circumferential strain (ϵ_y) was found as:

$$\sigma_z = \left[\frac{T}{c_1'} \right]^{-c_2'} - \left[\frac{T}{p_1'} \right]^{-p_2'} t \frac{\epsilon_y}{\frac{\partial \epsilon_y}{\partial t}}$$

Assuming the material to be isotropic and homogeneous, the following equations can be written:

$$\sigma_y = \left[\frac{T}{c_1} \right]^{-c_2} - \left[\frac{T}{p_1} \right]^{-p_2} t \frac{\epsilon_y}{\frac{\partial \epsilon_y}{\partial t}} = \left[\frac{T}{c_1'} \right]^{-c_2'} - \left[\frac{T}{p_1'} \right]^{-p_2'} t \frac{\epsilon_z}{\frac{\partial \epsilon_z}{\partial t}}$$

From the Simple Shear test results, the expression relating the shear stress (τ_{yz}) to shear strain (γ_{yz}), was found to be of similar algebraic form; namely,

$$\tau_{yz} = \left[\frac{T}{c_1''} \right]^{-c_2''} - \left[\frac{T}{p_1''} \right]^{-p_2''} t \frac{\gamma_{yz}}{\frac{\partial \gamma_{yz}}{\partial t}}$$

where c_1'' , c_2'' , p_1'' and p_2'' are material constants. The values of the four material constants as determined from Uniaxial Tension test results are:

$$\begin{array}{ll} c_1 = 130 & c_2 = 5.15 \\ p_1 = 98 & p_2 = 6.00 \end{array}$$

Y. 1 = 6 22 = 12

The corresponding material constants as determined from the Simple Shear test results are:

$$\begin{array}{ll} c_1'' = 150 & c_2'' = 4.40 \\ p_1'' = 108 & p_2'' = 5.15 \end{array}$$

A comparison of these material constants as obtained from the two types of tests show that they are quite close, considering the experimental limitations involved in the study. The stress-strain expressions from these two types of tests show that there are at least four material constants independent of time and temperature. These expressions when used to predict strains in an Axial Compression test gave reasonably good results for small strains up to 0.4% only, as observed in the previous section. For strains greater than 0.4% it appears that a different deformation mechanism is operating in compression as compared to tension tests. However, it must be recognized that pure compression was probably not achieved in the test performed and that the measurements made were less than ideal.

It may be pointed out here that these expressions can be usefully employed in devising a laboratory test for bituminous mixtures which would evaluate the constants for the material. Such a test would be quantitative and not just qualitative like the triaxial test used for testing bituminous mixtures.

The existence of at least four material constants, independent of time and temperature, as obtained from two different types of tests in this study, gives promise of more meaningful quantitative evaluation in the mixtures. The precise purpose of this study, as stated in the outline of the investigation was to determine more meaningful properties for a sheet-asphalt mixture than those customarily used. This has been achieved with the three stress-strain expressions

and the other two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

The first two are in the same position as the first.

obtained from relevant experimental data, which, with the two equations of motion in two dimensions, give a set of five equations and five unknowns as follows:

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} = z + m \frac{\partial^2 w}{\partial t^2} \dots \dots \dots (a)$$

$$\frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \sigma_y}{\partial y} = m \frac{\partial^2 v}{\partial t^2} \dots \dots \dots (b)$$

$$\begin{aligned} \sigma_z &= \left[\frac{T}{c_1} \right]^{-c_2} - \left[\frac{T}{p_1} \right]^{-p_2} \frac{\frac{\partial w}{\partial z}}{t \frac{\partial^2 w}{\partial t \partial z}} \\ &= \left[\frac{T}{c_1} \right]^{-c_2'} - \left[\frac{T}{p_1} \right]^{-p_2'} \frac{\frac{\partial v}{\partial y}}{t \frac{\partial^2 v}{\partial t \partial y}} \dots \dots \dots (c) \end{aligned}$$

$$\begin{aligned} \sigma_y &= \left[\frac{T}{c_1} \right]^{-c_2} - \left[\frac{T}{p_1} \right]^{-p_2} \frac{\frac{\partial v}{\partial y}}{t \frac{\partial^2 v}{\partial t \partial y}} \\ &= \left[\frac{T}{c_1} \right]^{-c_2'} - \left[\frac{T}{p_1} \right]^{-p_2'} \frac{\frac{\partial w}{\partial z}}{t \frac{\partial^2 w}{\partial z \partial t}} \dots \dots \dots (d) \end{aligned}$$

$$\tau_{yz} = \left[\frac{T}{c_1} \right]^{-c_2''} - \left[\frac{T}{p_1} \right]^{-p_2''} \frac{\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}}{t \left[\frac{\partial^2 w}{\partial t \partial y} + \frac{\partial^2 v}{\partial t \partial z} \right]} \dots \dots \dots (e)$$

It must be recognized, however, that the three stress-strain expressions obtained from experimental data are valid only for the range of temperatures and stress-levels for which the material was tested in this study.

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

It is shown that the function $f(x)$ is continuous and differentiable on the interval $(0, \infty)$.

$$f'(x) = -\frac{1}{x^2} \int_0^x f(t) dt + \frac{1}{x} f(x)$$

The second part of the paper is devoted to the study of the properties of the function $g(x)$ defined by the equation

$$g(x) = \frac{1}{x} \int_0^x g(t) dt + \frac{1}{x^2} \int_0^x t g(t) dt$$

It is shown that the function $g(x)$ is continuous and differentiable on the interval $(0, \infty)$.

$$g'(x) = -\frac{1}{x^2} \int_0^x g(t) dt - \frac{1}{x^3} \int_0^x t g(t) dt + \frac{1}{x} g(x) + \frac{1}{x^2} g(x)$$

The third part of the paper is devoted to the study of the properties of the function $h(x)$ defined by the equation

CONCLUSIONS

The following conclusions have been drawn from the experimental data obtained for the sheet-asphalt mixture, within the range of temperatures and stress-levels for which it was tested in this investigation:

1. Three independent stress-strain relationships exist as functions of time and temperature which together with the two-dimensional equations of motion, give a system of five equations containing five unknowns.

2. For the sheet asphalt mixture tested, there exist at least four basic material constants independent of time and temperature as opposed to the usual modulus of elasticity and Poisson's ratio constants assumed in elastic theory. These four basic material constants exist in the tensile stress-axial strain expression derived from Uniaxial Tension test results and also in the shear stress-shear strain expression derived from Simple Shear test results. From the fact that the magnitude of the material constants as determined from two different types of tests, performed for a number of different conditions of time and temperature, were quite close to each other, it may be concluded that these material constants are independent of the type of test. As the results from Axial Compression tests corresponded reasonably well with those predicted from Uniaxial Tension test results for strains less than about 0.4 percent, it may be concluded that the derived expressions hold for both tension and compression of the material for very small strains.

REFERENCES

1. Pister, K. E. and Monismith, C. L., "Analysis of Visco-Elastic Flexible Pavements," Flexible Pavement Design Studies, Highway Research Board Bulletin 269, 1960.
2. Monismith, C. L. and Secor, K. E., "Visco-Elastic Behavior of Asphalt Concrete Pavements," International Conference on the Structural Design of Asphalt Pavements, University of Michigan, Ann Arbor, August 1962.
3. Wood, L. E. and Goetz, W. H., "Rheological Characteristics of a Sand-Asphalt Mixture," Proceedings, Association of Asphalt Paving Technologists, Vol. 28, 1959.
4. Secor, K. E. and Monismith, C. L., "Analysis of Triaxial Test Data on Asphalt Concrete Using Visco-Elastic Principles," Proceedings, Highway Research Board, Vol. 40, 1961.
5. Secor, K. E. and Monismith, C. L., "Visco-Elastic Properties of Asphalt Concrete," Proceedings, Highway Research Board, Vol. 41, 1962.
6. Davis, E. F., Krokosky, E. M. and Tons, E., "Stress Relaxation of Bituminous Concrete in Tension," MIT Report, R63-40, Massachusetts Institute of Technology, August 1963.
7. Lal, N. B., "Two Dimensional Stress-Strain Relationships of a Fine Aggregate-Asphalt System," A Thesis submitted to Purdue University in partial fulfillment of the requirements for the Ph. D. degree, May 1965.

1. The first of these is the fact that the
the first of these is the fact that the
the first of these is the fact that the

2. The second of these is the fact that the
the second of these is the fact that the
the second of these is the fact that the

3. The third of these is the fact that the
the third of these is the fact that the
the third of these is the fact that the

4. The fourth of these is the fact that the
the fourth of these is the fact that the
the fourth of these is the fact that the

5. The fifth of these is the fact that the
the fifth of these is the fact that the
the fifth of these is the fact that the

6. The sixth of these is the fact that the
the sixth of these is the fact that the
the sixth of these is the fact that the

7. The seventh of these is the fact that the
the seventh of these is the fact that the
the seventh of these is the fact that the

TABLE 1

Axial Strain - Time Relationships

For

Uniaxial Tension Tests

40° F				77° F				100° F			
Applied Tensile Stress	Axial Strain at One-Minute	Slope of Axial Strain vs. Time Plot (log-log)	Applied Tensile Stress	Axial Strain at One-Minute	Slope of Axial Strain vs. Time Plot (log-log)	Applied Tensile Stress	Axial Strain at One-Minute	Applied Tensile Stress	Axial Strain at One-Minute	Slope of Axial Strain vs. Time Plot (log-log)	
Psi	0.0001 in./in.		Psi	0.0001 in./in.		Psi	0.0001 in./in.	Psi	0.0001 in./in.		
18.43	2.55	1:1.95	1.70	11.5	1:2.80	0.75	7.2	1:4.40			
30.43	4.40	1:1.90	2.43	15.0	1:2.50	1.07	18.5	1:3.70			
40.43	6.60	1:1.85	4.43	28.5	1:2.00	1.43	26.0	1:3.20			
52.43	9.60	1:1.80	5.43	39.0	1:1.80	2.43	60.0	1:2.40			

—

100



10

•

1

1

20

•

•

•

10

•

100

2. 100

9

4

•

•

..

TABLE 2

Circumferential Strain - Time Relationships

For

Uniaxial Tension Tests

40° F				77° F				100° F			
Applied Tensile Stress	Circum- ferential Strain at One-Minute 0.0001 in./in.	Slope of Circum- ferential Strain vs. Time Plot (log-log)	Applied Tensile Stress	Circum- ferential Strain at One-Minute 0.0001 in./in.	Slope of Circum- ferential Strain vs. Time Plot (log-log)	Applied Tensile Stress	Circum- ferential Strain at One-Minute 0.0001 in./in.	Slope of Circum- ferential Strain vs. Time Plot (log-log)	Applied Tensile Stress	Circum- ferential Strain at One-Minute 0.0001 in./in.	Slope of Circum- ferential Strain vs. Time Plot (log-log)
18.43	1.18	1:2.00	1.70	4.25	1:2.40	0.75	3.25	1:3.00			
30.43	1.75	1:1.95	2.43	5.50	1:2.30	1.07	6.5	1:2.85			
40.43	2.75	1:1.90	4.43	8.50	1:2.20	1.43	10.0	1:2.75			
52.43	3.75	1:1.85	5.43	11.0	1:2.12	2.43	20.0	1:2.50			

TABLE 3

Shear Strain - Time RelationshipsForSimple Shear Tests

40° F				77° F				100° F			
Applied Shear Stress Psi	Shear Strain at One-Minute 0.0001 in./in.	Slope of Shear Strain vs. Time Plot (log-log)	Applied Shear Stress Psi	Shear Strain at One-Minute 0.0001 in./in.	Slope of Shear Strain vs. Time Plot (log-log)	Applied Shear Stress Psi	Shear Strain at One-Minute 0.0001 in./in.	Applied Shear Stress Psi	Shear Strain at One-Minute 0.0001 in./in.	Slope of Shear Strain vs. Time Plot (log-log)	Slope of Shear Strain vs. Time Plot (log-log)
5.64	8.0	1:2.55	1.73	30	1:3.00	0.563	34.0			1:4.50	
10.05	13.6	1:2.50	3.32	56.0	1:2.80	0.950	56.0			1:4.00	
18.3	24.4	1:2.45	4.83	80.0	1:2.60	1.35	88.0			1:3.50	
23.82	32.0	1:2.40	6.45	114.0	1:2.50	1.732	110.0			1:3.38	
			8.02	146.0	1:2.30						

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

1911

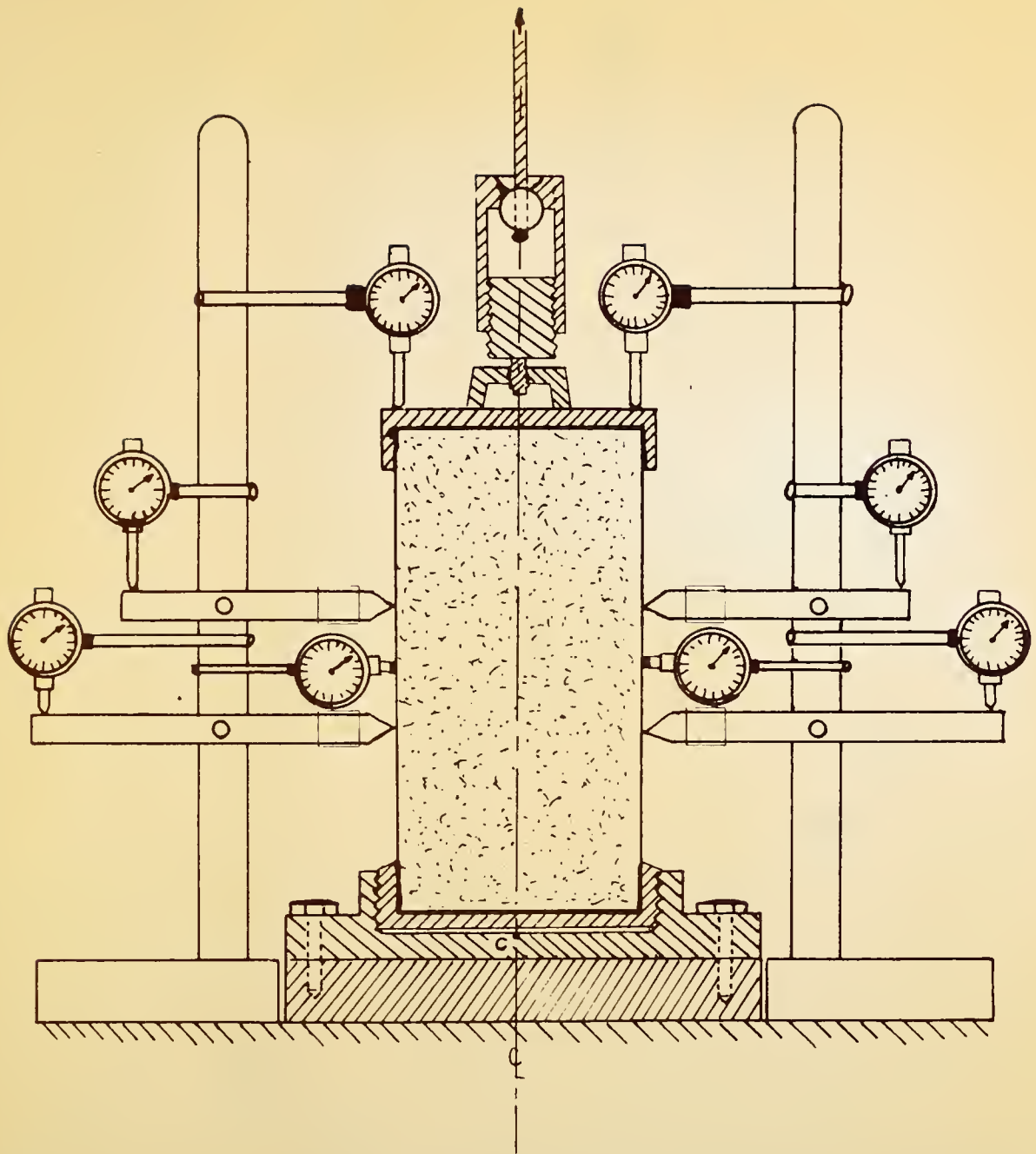


Figure 1 Diagrammatic Sketch Showing Instrumentation of a Uniaxial Tension Test.

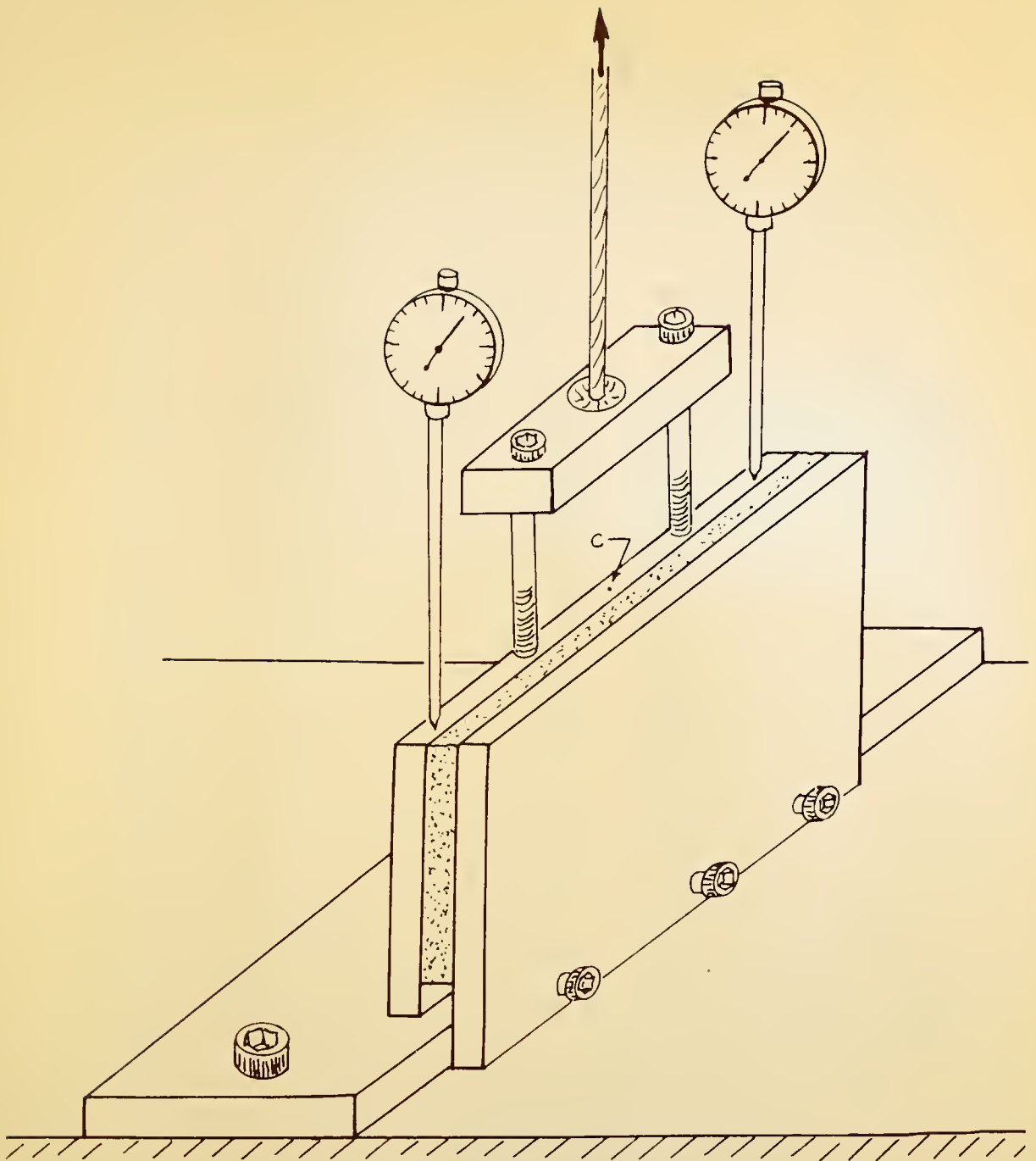


Figure 2 Diagrammatic Sketch Showing Instrumentation of a Simple Shear Test.

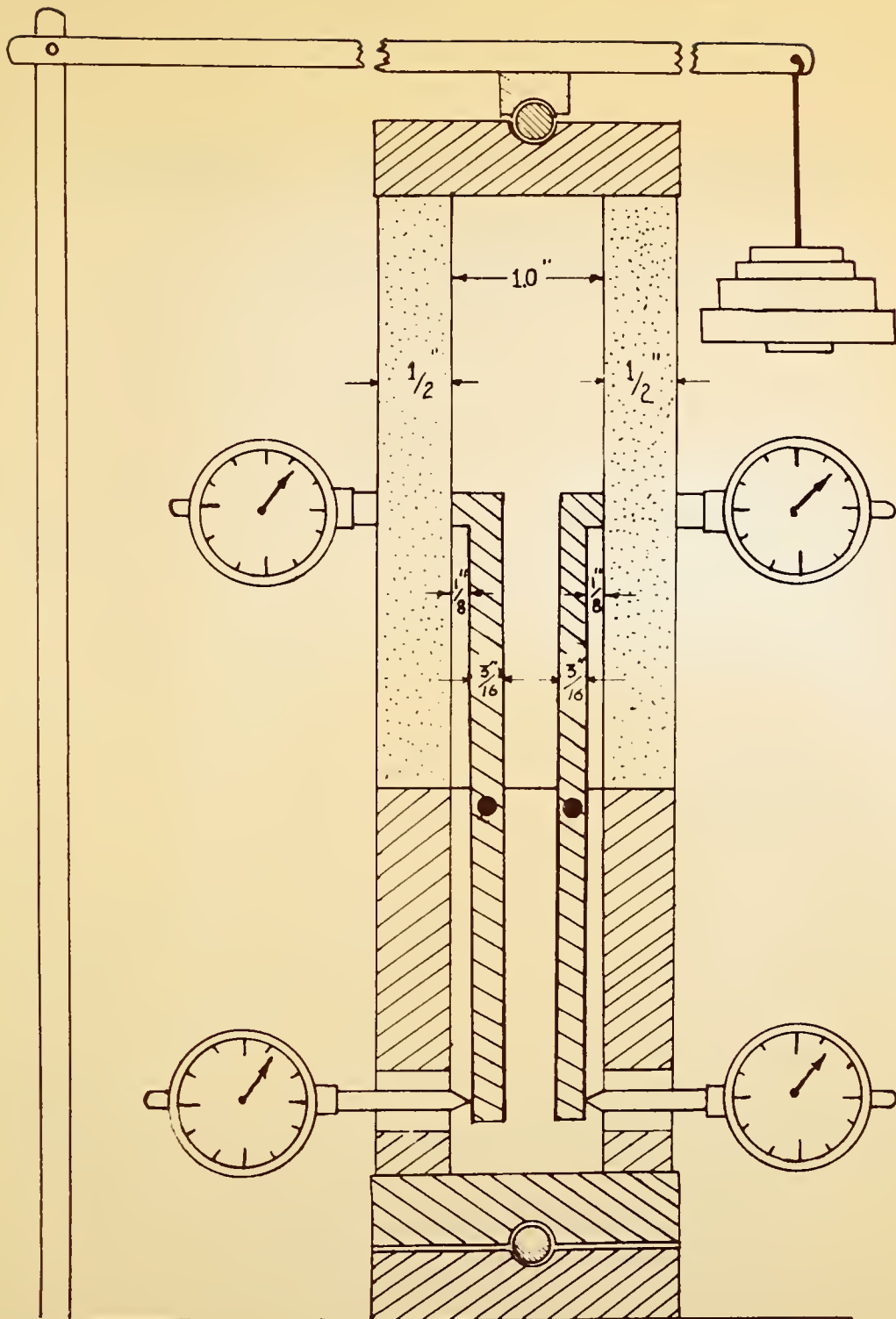


Figure 3 Diagrammatic Sketch Showing Instrumentation for Change in Thickness Measurements in Axial Compression Test.

UNIAXIAL TENSION TEST RESULTS

AXIAL STRAIN vs. TIME CURVES

TEMPERATURE = 77°F

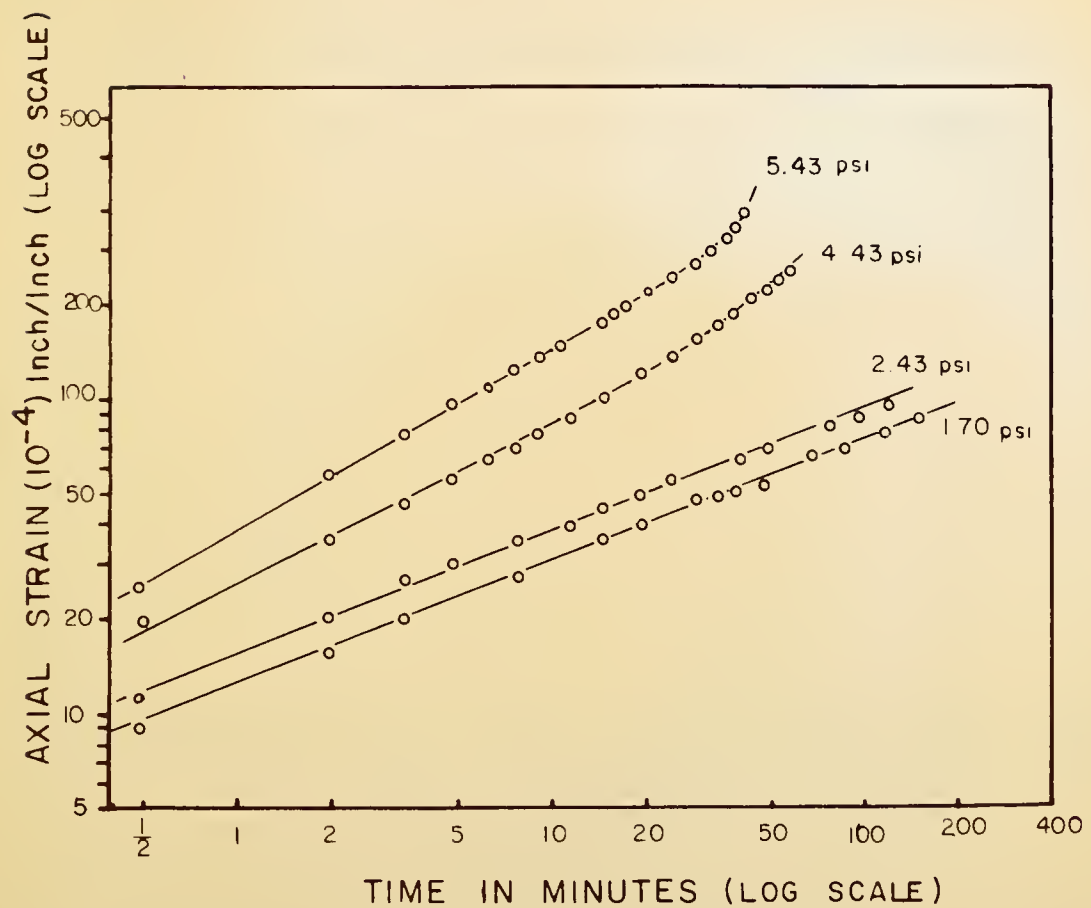


FIG. 4

UNIAXIAL TENSION TEST RESULTS

CIRCUMFERENTIAL STRAIN vs.
TIME CURVES

TEMPERATURE = 77°F

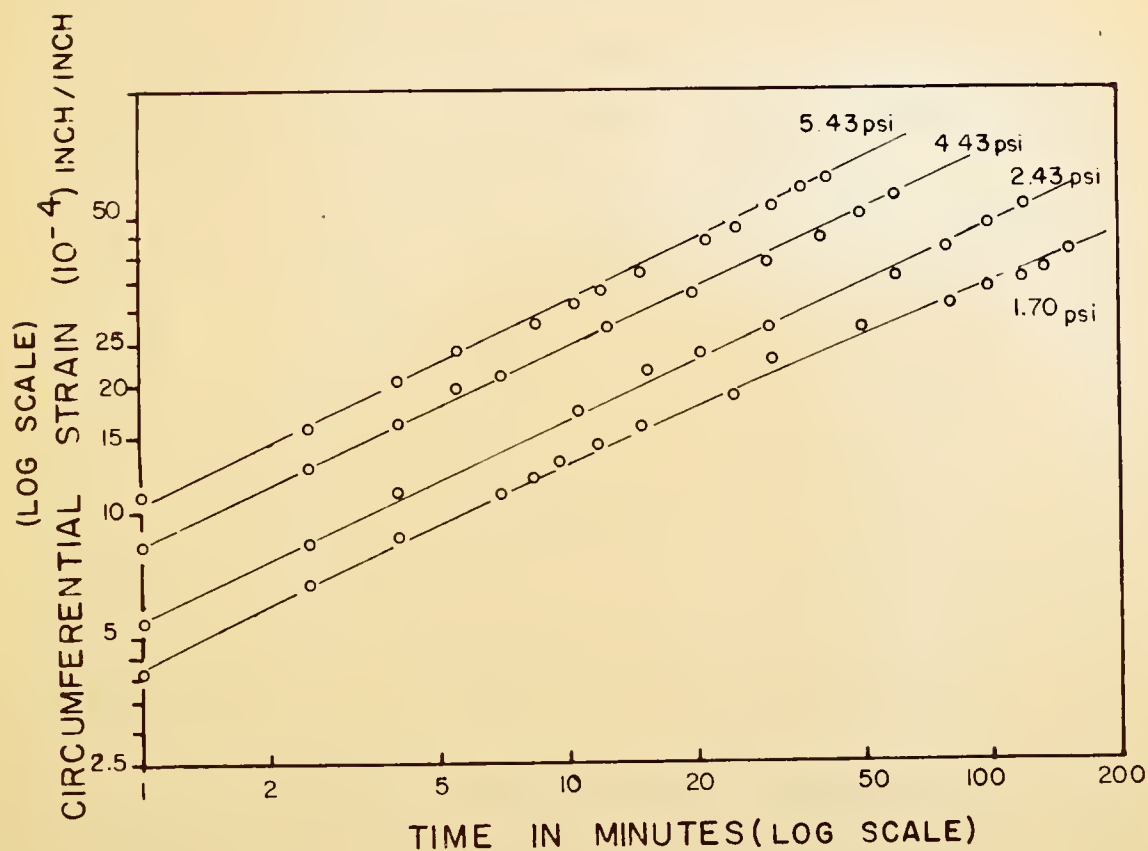


FIG. 5

SIMPLE SHEAR TEST RESULTS

SHEAR STRAIN vs. TIME CURVES

TEMPERATURE = 77°F

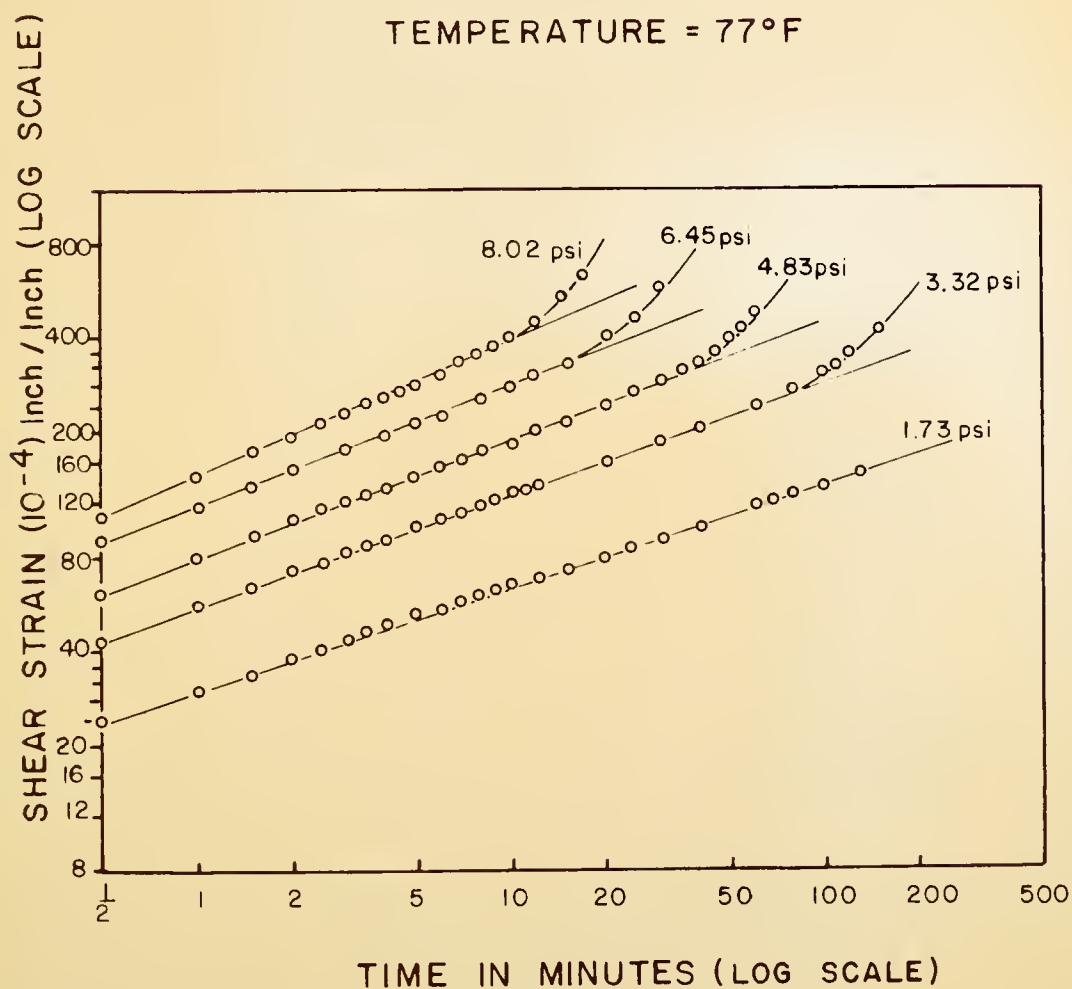


FIG. 6

AXIAL COMPRESSION TEST RESULTS

AXIAL STRAIN vs. TIME
CURVES

TEMPERATURE = 77°F

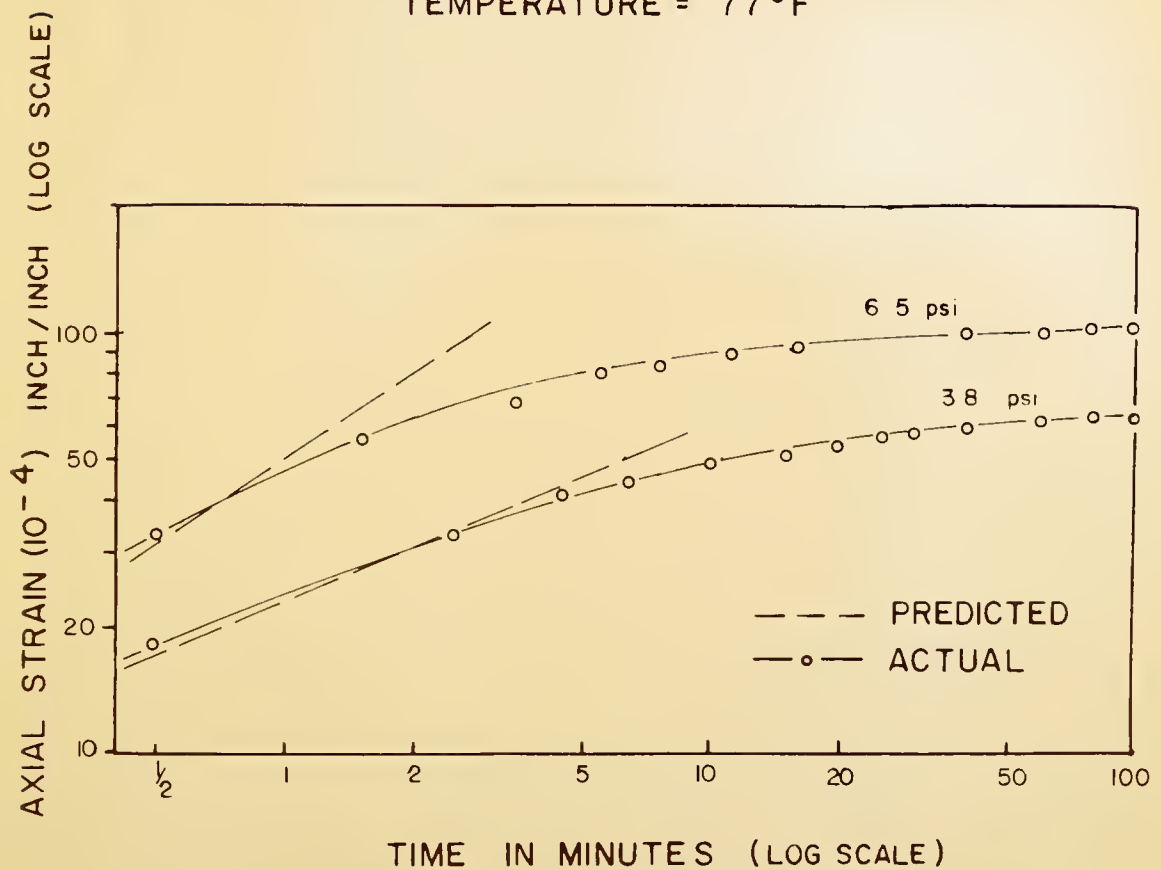


FIG. 7

AXIAL COMPRESSION TEST RESULTS

CIRCUMFERENTIAL STRAIN vs. TIME
CURVES

TEMPERATURE = 77°F.

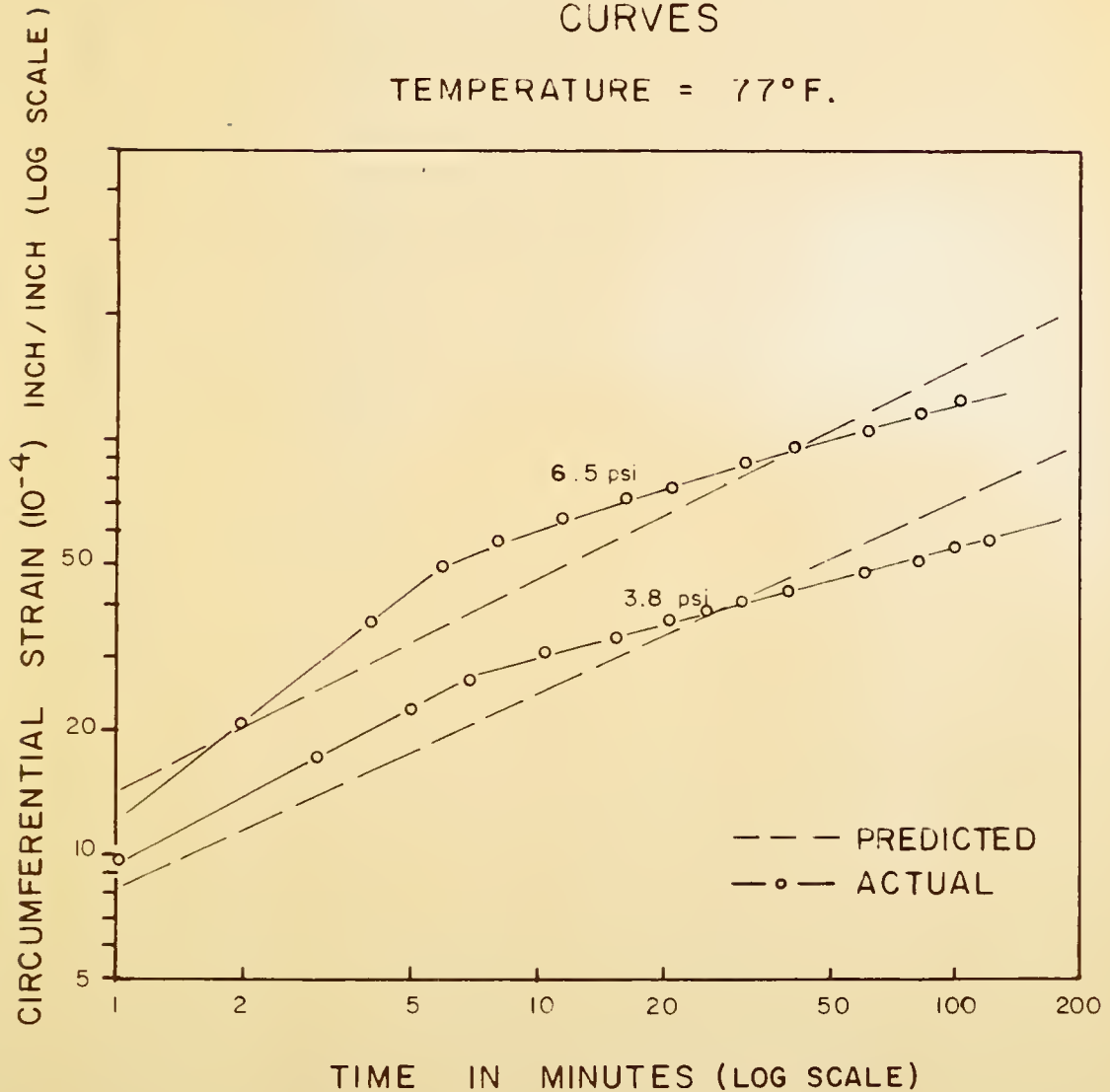


FIG. 8

UNIAXIAL TENSION TEST RESULTS

$1/K_2$ vs. σ_z

$1/K_2$ = RECIPROCAL OF SLOPE OF AXIAL STRAIN vs. TIME
LOG-LOG PLOT

σ_z = TENSILE STRESS IN PSI

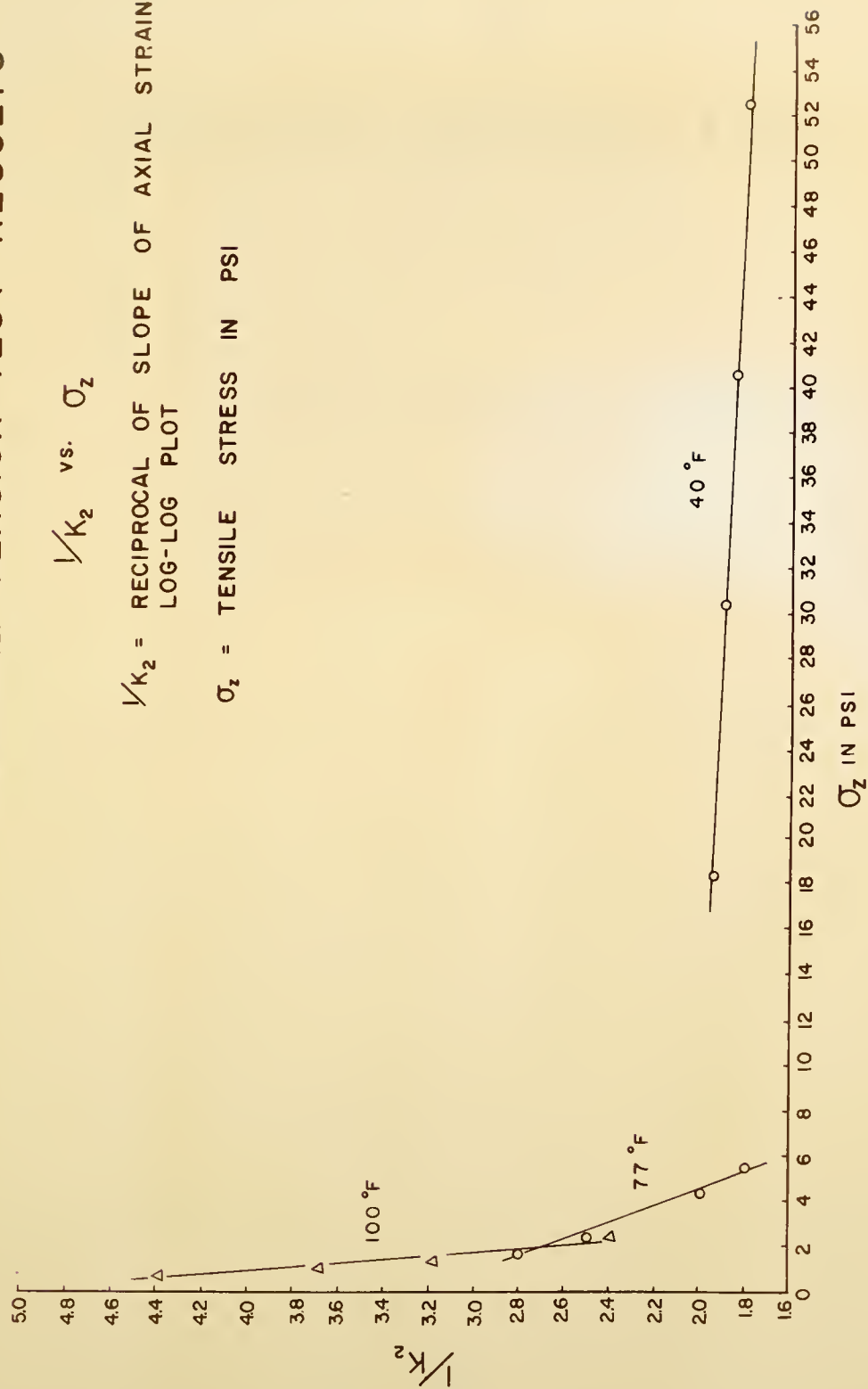


FIG. 9

UNIAXIAL TENSION TEST RESULTS

$\text{LOG} [I(T)/S(T)] \text{ vs. } \text{LOG}(T)$

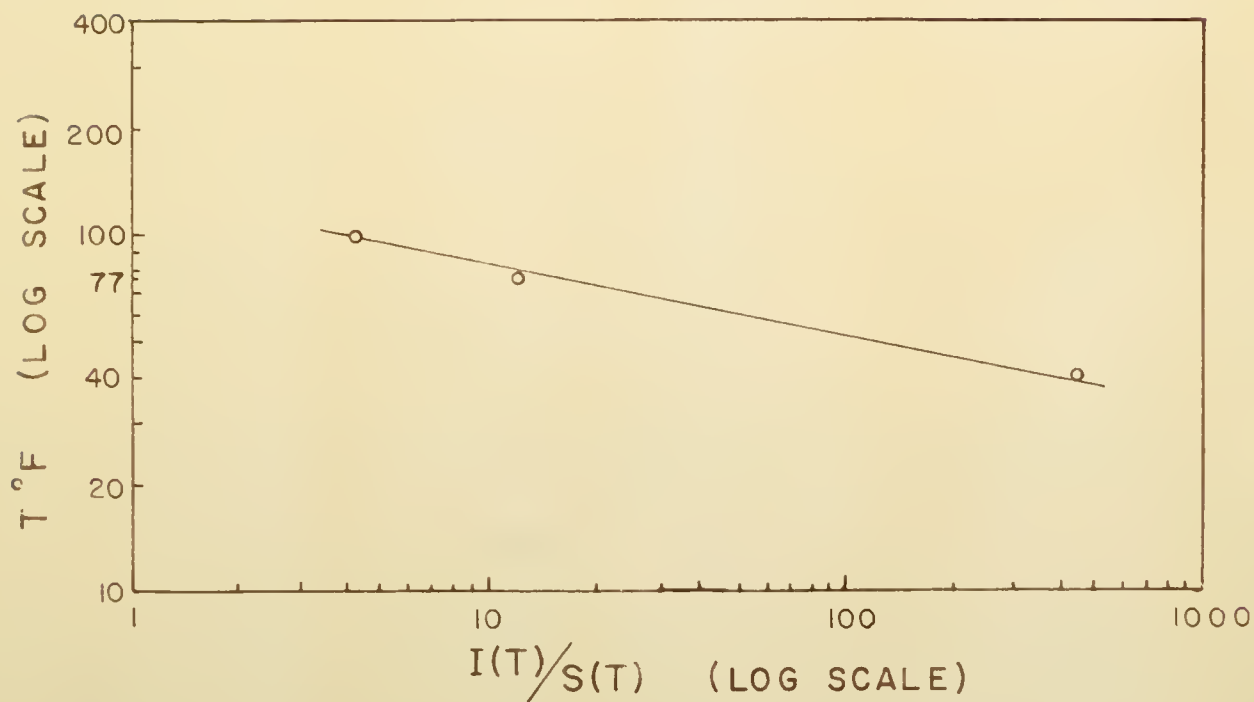


FIG. 10

UNIAXIAL TENSION TEST RESULTS

$\text{LOG} \left[\frac{l}{S(T)} \right]$ vs. $\text{LOG}(T)$

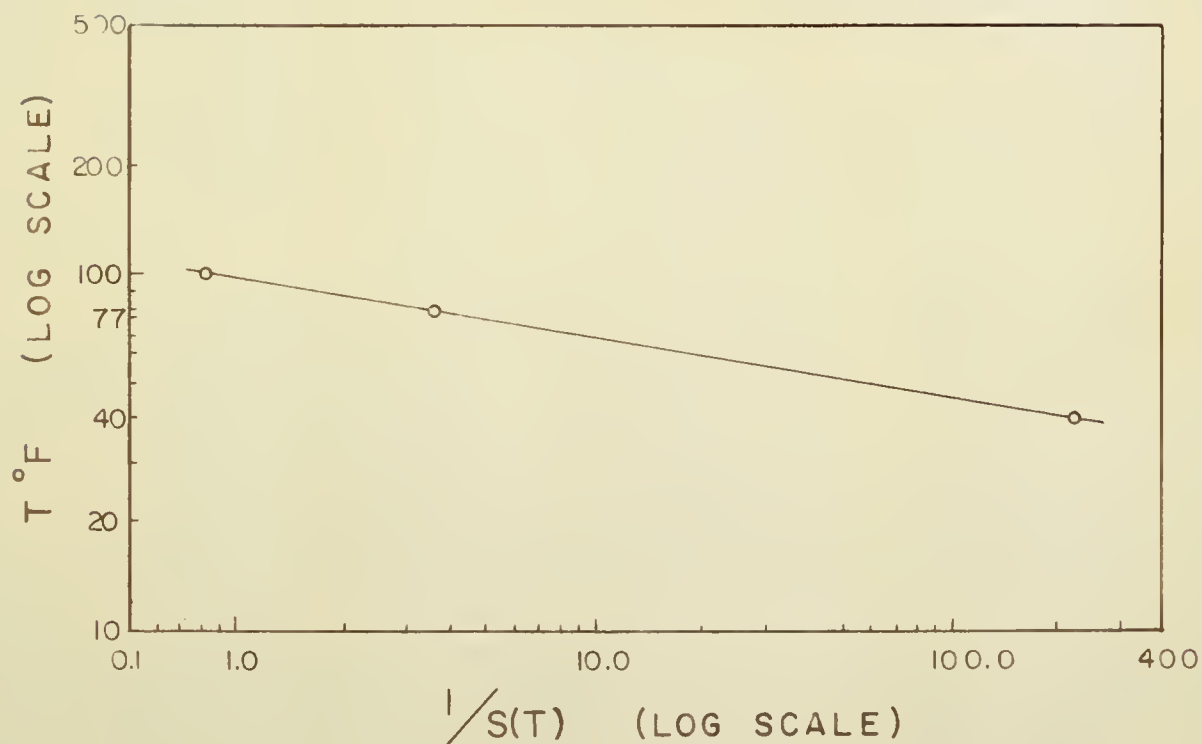


FIG. II

